Pre-AP Precalculus Semester Review
Spring 2016

You may make a notecard for the final exam. It must be no larger than a 3x5 card, no computer-generated, typed, or photocopied cards. You may write on the front and back.

Due to space limitations, most of these problems should be worked on separate paper.

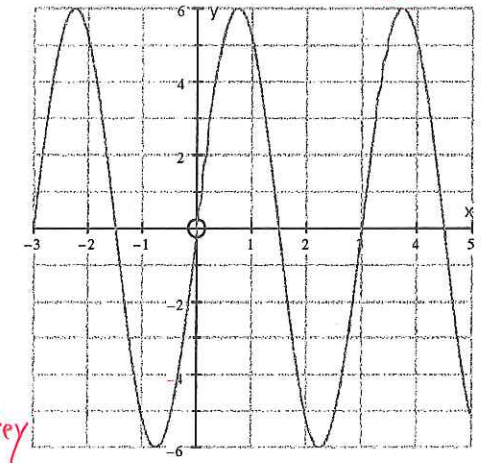
Key

Solve for $0 \le x < 2\pi$. Give the solutions in terms of π.

1. $\sqrt{2}\cos x - 1 = 0$ $\cos x = \frac{\sqrt{2}}{2}$ $x = \frac{\pi}{4}, \frac{7\pi}{4}$

2. $\sqrt{\sec^2 x} = \sqrt{4}$ $\frac{1}{\cos x} = \frac{2}{1}$ $\sec x = 2$ $\cos x = \pm\frac{1}{2}$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$

In #3 – 6, refer to the given graph.



3. Find the amplitude. 6

4. Find the period. $P = \frac{2\pi}{B}$ $P = \frac{2\pi}{3}$

5. Find an equation of the curve. $y = 6\sin\left(\frac{2\pi}{3}\right)x$

6. Sketch $y = -4 + 2\cos 3(x - \pi)$. amp → 2, period: $\frac{2\pi}{3}$, H.S. → π right, V.S. → 4 down *Graph on last page of key

Simplify the expression.

7. (1-sin x) (1+ csc x) sin x $(1 + \csc x - \sin x - \sin x \csc x)\sin x$

$\sin x + \frac{1}{\sin x}\cdot\sin x - \sin^2 x - \sin^2 x \cdot \frac{1}{\sin x}$

$\sin x + 1 - \sin^2 x - \sin x$

$1 - \sin^2 x = \cos^2 x$

Solve for $0° \le x < 360°$. Where necessary, round the solution to the nearest tenth of a degree.

8. $3\cos\theta - 4\sin\theta = 0$ $\frac{3\cos\theta}{\cos\theta} = \frac{4\sin\theta}{\cos\theta}$ $+4\sin\theta \quad +4\sin\theta$ $3 = 4\tan\theta$ $\frac{3}{4} = \tan\theta$ $\theta = 36.9°, 216.870$

9. $9\sin\theta = \csc\theta$ $9\sin\theta = \frac{1}{\sin\theta}$ $\sqrt{\sin^2\theta} = \sqrt{\frac{1}{9}}$ $\sin\theta = \pm\frac{1}{3}$ $\theta = 19.5°, 160.529°, 199.471°, 340.529°$

Determine whether each of the following (18-20) is always, sometimes, or never true.

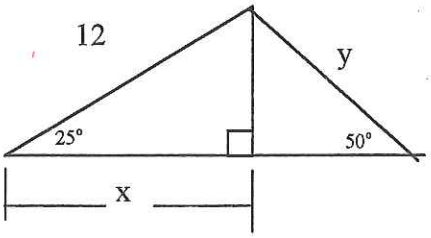
10. In $\triangle$ ABC with right angle C, sin A = cos B. Always

11. An angle of depression or elevation is the angle between the line of sight and the horizontal.

Always

12. The compass bearing of the course from one location to another is the angle from the north direction clockwise to the other location.

Always



13. Find x and y. Refer to the figure above.

Law of sines

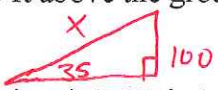
X: $\cos 25 = \frac{x}{12}$ $x = 12\cos 25$ $x = 10.876$

Y: $\frac{\sin 25}{y} = \frac{\sin 50}{12}$ $y = 6.620$

14. The sides of an isosceles triangle have lengths 10, 10, and 4. Find the measure of its base angles.

$\theta = \cos^{-1}\left(\frac{2}{10}\right)$ $\theta = 78.463$

15. A kite is 100 ft above the ground. How many feet of string are used if the angle of elevation is 35°.



$\sin 35 = \frac{100}{x}$ $x = 174.345$ ft

16. The area of triangle PQR is 24. If p = 6 and q = 8, find the measure of angle R.

$A = \frac{1}{2}ab\sin C$ $24 = \frac{1}{2}(6)(8)\sin R$ $\angle R = 90°$

17. In triangle ABC, a = 6 , b = 7, cos C = ¼.

a) Find the exact area of the triangle.

$A = \frac{1}{2}(6)(7)(\frac{1}{4})$

$A = 21/4$

b) Find side c.

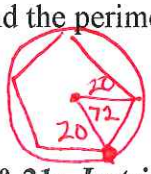
$c^2 = 6^2 + 7^2 - 2(6)(7)\cos C$

$c^2 = 64$ $c = 8$

18. In triangle XYZ, $\angle X = 30°, \angle Y = 50°, and\ z = 10$. Find side x.

$\frac{\sin 100}{10} = \frac{\sin 30}{x}$ $x = 5.077$

19. Find the perimeter of a regular pentagon inscribed in a circle with radius 20.



$x^2 = 20^2 + 20^2 - 2(20)(20)\cos 72$

$x = 23.511$ $P = 5x$ $P = 117.557$

For #20-21: In triangle RST, r = 6, s = 10, t = 14.

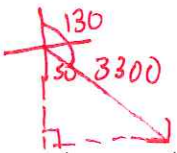
20. Find the measure of the largest angle to the nearest degree.

$14^2 = 6^2 + 10^2 - 2(6)(10)\cos T$ $\angle T = 120°$

21. Find the length of the altitude from S.

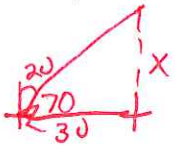
$\sin 60 = \frac{h}{6}$ $h = 5.196$

22. An airplane flies on a course of 130° at a speed of 1100 km/h. How far east of its starting point is it after 3 hours?



$\sin 50 = \frac{x}{3300}$ $x = 2527.947$ km

23. A ship leaves port and proceeds west 30 miles. It then changes course to 020° until it is due north of its origin. How far north of its origin is it? (Round to the nearest mile.)



$\tan 70 = \frac{x}{30}$ $x = 82.424$ mi.

24. Use a sum formula to find $\sin 75°$ in simplest radical form.

25. Simplify: $\sin x \cos(x + y) - \cos x \sin(x + y)$

26. Simplify: $\cos(\pi/3 + x) + \cos(\pi/3 - x)$

24-33 on last page of key!

27. Suppose $\angle A$ is acute and $\cos A = 4/5$. Find $\sin 2A$.

28. Simplify: $\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$

29. Simplify: $2\cos^2(3A) - 1$

30. Simplify: $\dfrac{\sin 2x}{1+\cos 2x}$

31. Find the component form and magnitude of the vector. $u = \overrightarrow{AB}$, $A = (4, 2)$ and $B = (-9, -1)$.

32. Find $-2\mathbf{u} + 5\mathbf{v}$, if $\mathbf{u} = \langle 6, -2 \rangle$ and $\mathbf{v} = \langle 3, -5 \rangle$.

33. Write out the general parametric equations for a circle, ellipse, and parabola.
Know how to graph a set of parametric equations in your calculator.
Also know how to graph a polar equation.

34. Refer to the ellipse represented by $\frac{x^2}{9} + \frac{(y-1)^2}{4} = 1$. $a^2 \; b^2$

a) Find the coordinates of the center. C: (0, 1)

b) Find the coordinates of the foci. $c^2 = a^2 - b^2$ foci: $(\pm\sqrt{65}, 1)$

$c^2 = 81 - 16 \quad c = \sqrt{65}$

$35, \cos\theta = -x \quad \sin\theta = y$

$x^2 + y^2 = 1$

35. Write the pair of parametric equations $x = -\cos\theta$ and $y = \sin\theta$ in rectangular form.

36. Rusty hit a baseball with initial velocity of 125 feet per second at an angle of 32° with the ground from a height of 3 feet. Write parametric equations to represent this situation.

F $x = 125t \cos 32° - 16t^2$
$y = 125t \sin 32° + 3$

G $x = 32 \cos 125t° + 3$
$y = 32 \cos 125t°$

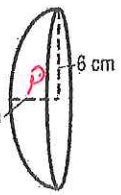
H $x = 32t \cos 125°$
$y = 32t \sin 125° - 16t^2$

J $x = 125t \cos 32°$
$y = 125t \sin 32° - 16t^2 + 3$

$x = 125 \cos 32t$

$y = 125 \sin 32t - 16t^2 + 3$

37. A cross section of the reflector shown is in the shape of a parabola. Write an equation for the cross section.



$p = 3$

$a = \frac{1}{4(3)} = \frac{1}{12}$

A $y^2 = 12x$
B $y^2 = 8x$
C $x^2 = 4y$
D $x^2 = 8y$

$x = \frac{1}{12} y^2$

$12x = y^2$

38. Refer to the hyperbola represented by $\frac{(x-3)^2}{16} - \frac{y^2}{9} = 1$. ><

a) Write the equations of the asymptotes.

$\pm\frac{b}{a} \quad \pm\frac{3}{4}$

b) Find the coordinates of the foci.

$c^2 = a^2 + b^2$

$c^2 = 3^2 + 4^2$ $c = 5$

C: (3, 0) foci: (−2, 0) and (8, 0) goes through

39. Write the standard form of the equation of the hyperbola for which $a = 5$, $b = 6$, the transverse axis is vertical, and the center is at the origin.

F $\frac{y^2}{25} - \frac{x^2}{36} = 1$ G $\frac{x^2}{36} - \frac{y^2}{25} = 1$ H $\frac{x^2}{25} - \frac{y^2}{36} = 1$ J $\frac{y^2}{36} - \frac{x^2}{25} = 1$

a^2 b^2

40. Jana hit a golf ball with an initial velocity of 102 feet per second at an angle of 67° with the horizontal. After 2 seconds, how far has the ball traveled horizontally?

A 27.9 ft B 123.8 ft C 79.7 ft D 97.7 ft

$x = v_0 \cos 67 \cdot t$

$x = 102 \cos 67 \cdot 2$

41. Write the standard form of the equation of the parabola with directrix at $x = -2$ and focus at (2, 0).

$x = \frac{1}{8} y^2$

$p = 2$ $a = \frac{1}{4p} = \frac{1}{8}$ V: (0, 0) 2, 0

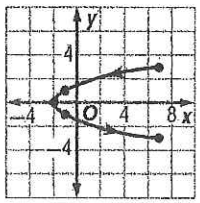
42. Identify the conic represented by $6x^2 - 8xy - 2y^2 + 6 = 0$.

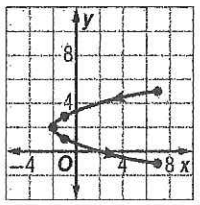
43. Which graph represents a curve given by $x = t - 2$ and $y = t^2 - 2$ over the interval $-3 \le t \le 3$?

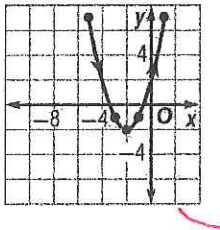
$t = x + 2$

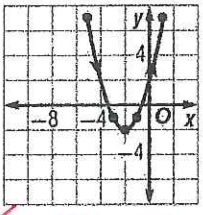
$y = (x+2)^2 - 2$

V: (−2, −2)

F 

G 

H 

J 

Do these look the same to you?

44. Write the standard form of the equation of the circle with center at (−4, 8) that is tangent to the x-axis.

F $x^2 + y^2 = 64$

G $(x + 4)^2 + (y - 8)^2 = 16$

H $(x - 4)^2 + (y + 8)^2 = 16$

J $(x + 4)^2 + (y - 8)^2 = 64$

LIMITS ARE RECENT, SO YOU DON'T NEED MORE OF THOSE PROBLEMS!
STUDY YOUR NOTES FROM LIMITS

These review problems are a sampling of the types of problems you worked on all semester. Please do not expect every exam problem to be just like these review problems. You should also go back over your class notes and old quizzes.

Don't forget: Make a formula card—One 3 X 5 index card, both sides, handwritten.

24. $\sin 75 = \sin(30+45) = \sin 30\cos 45 + \cos 30 \sin 45$

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$

25. $\sin x(\cos x\cos y - \sin x\sin y) - \cos x(\sin x\cos y + \cos x\sin y)$

$\sin x\cos x\cos y - \sin^2 x\sin y - \sin x\cos x\cos y - \cos^2 x\sin y$

$-\sin^2 x\sin y - \cos^2 x\sin y$

$-\sin y(\sin^2 x + \cos^2 x)$

1

$= -\sin y$

26. $\cos\frac{\pi}{3}\cos x - \sin\frac{\pi}{3}\sin x + \cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x$

$2\cos\frac{\pi}{3}\cos x$

$2(\frac{1}{2})\cos x$

$= \cos x$

27. $\cos A = \frac{4}{5}$ $\sin A = \frac{3}{5}$

$\sin 2A = 2\sin A\cos A$

$= 2(\frac{3}{5})(\frac{4}{5})$

$= 24/25$

28. $\cos^2(\frac{x}{2}) - \sin^2(\frac{x}{2})$

$= \cos 2(\frac{x}{2})$

$= \cos x$

29. $2\cos^2(3A) - 1$

$= \cos 2(3A)$

$= \cos 6A$

30. $\frac{\sin 2x}{1+\cos 2x}$

$= \frac{2\sin x\cos x}{1+2\cos^2 x - 1}$

$= \frac{2\sin x\cos x}{2\cos^2 x}$

$= \frac{\sin x}{\cos x} = \boxed{\tan x}$

31. $\langle -9, -1-2\rangle$

$\langle -13, -3\rangle$

$|\vec{v}| = \sqrt{(-13)^2+(-3)^2} = \sqrt{178}$

32. $-2u+5v$

$-2\langle 6,-2\rangle + 5\langle 3,-5\rangle$

$\langle -12, 4\rangle + \langle 15, -25\rangle$

$\langle 3, -21\rangle$

33. circle: $x = a\cos(t)$

$y = a\sin(t)$

ellipse: $x = a\cos(t)$

$y = b\sin(t)$

parabola: $x = a\cos^2(t)$

$y = b\sin(t)$

6.